J/ψ-Photoproduction
and the Gluon Structure of the Nucleon

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Abstract:

Using short distance QCD methods based on the operator product expansion, we calculate the J/ψ photoproduction cross section in terms of the gluon distribution function of the nucleon. Comparing the result with data, we show that experimental behaviour of the cross section correctly reflects the x-dependence of the gluon distribution obtained from deep inelastic scattering.

The charmonium ground state J/ψ is much smaller than the conventional hadrons constructed from light u and d quarks, and much more tightly bound: we recall that $r_{J/\psi} \simeq 0.2$ fm $\ll \Lambda_{QCD}^{-1}$ and $2M_D - M_{J/\psi} \simeq 0.64$ GeV $\gg \Lambda_{QCD}$. As a result, when a J/ψ interacts with a 'light' hadron, it is expected to probe the local partonic structure of the latter, not its size or mass. The aim of this paper is to show that measurements of J/ψ-photoproduction on nucleons confirm this expectation and give a rather precise reflection of the gluon distribution function of a nucleon.

Using vector meson dominance (VMD), we shall first relate forward J/ψ-photoproduction to J/ψ-nucleon scattering. Next, the total J/ψ-nucleon cross section is expressed in terms of nucleonic gluon distribution functions, making use of the short distance QCD methods based on sum rules derived from the operator product expansion (OPE). The last missing element, the ratio of real and imaginary $J/\psi-N$ scattering amplitudes, is obtained by a dispersion relation. We can then study directly the interrelation of the energy dependence of J/ψ-photoproduction and the x dependence of the gluon distribution in the nucleon.

The use of the J/ψ as a probe of the partonic status of a given medium is of particular interest for the study of colour deconfinement in nucleus-nucleus collisions. An essential and here particularly relevant aspect of deconfinement is that the constituent partons of a deconfined medium are no longer constrained to the distribution functions of individual hadrons, as determined in deep inelastic scattering. The hadronic gluon distribution functions are strongly suppressed at high gluon momenta, with $xg(x) \sim (1-x)^a$ for $x \to 1$, where $a \simeq 3 - 5$ and $x = k_g/k_h$ is the relative fraction of the hadron momentum carried by the gluon. Removing this constraint will generally lead to harder gluons. Since J/ψ dissociation requires hard gluons, the inelastic J/ψ-hadron cross section becomes very small at low collision energies. Hence significant J/ψ suppression in nuclear collisions requires the produced environment to be deconfined.

In principle, the predicted threshold suppression of the J/ψ-hadron dissociation cross section can be measured directly. However, until such experiments are carried out, and to check the universality of the phenomenon, it is of interest to consider other processes in which the partonic aspects of the J/ψ-hadron interactions play a decisive role. The photoproduction of heavy quarkonium states is, as we shall see, an excellent case at hand, with a quite extensive set of experimental data available over a wide region of incident energies.

Within the conventional VMD approach, we can relate the reactions $\gamma N \to \psi N$ and $\psi N \to \psi N$ and express the differential cross section of the forward J/ψ-photoproduction on nucleons as [1]

$$\frac{d\,\sigma_{\gamma N \to \psi N}}{d\,t}(s, t = 0) = \frac{3\Gamma(\psi \to e^+ e^-)}{\alpha m_\psi} \left(\frac{k_{\psi N}}{k_{\gamma N}}\right)^2 \frac{d\,\sigma_{\psi N \to \psi N}}{d\,t}(s, t = 0) \tag{1}$$

where $k_{ab}^2 = [s - (m_a + m_b)^2][s - (m_a - m_b)^2]/4s$ denotes the squared center of mass momentum of the corresponding reaction and Γ stands for the partial J/ψ-decay width; to simplify formulae, we shall in most expressions abbreviate J/ψ by ψ. The differential $J/\psi - N$ cross section is given by

$$\frac{d\,\sigma_{\psi N \to \psi N}}{d\,t}(s, t = 0) = \frac{1}{64\pi} \frac{1}{m_\psi^2(\lambda^2 - m_N^2)} |\mathcal{M}_{\psi N}(s, t = 0)|^2 \tag{2}$$

with $\mathcal{M}_{\psi N}$ denoting the invariant $J/\psi - N$ scattering amplitude; further, $\lambda = (pK/m_\psi)$ is the nucleon energy in the quarkonium rest frame and p, K, q are the four-momenta of target nucleon, J/ψ and initial photon, respectively. From the optical theorem in the form

$$\sigma_{\psi N}^{tot} = \frac{Im\,\mathcal{M}_{\psi N}}{2m_\psi \sqrt{\lambda^2 - m_N^2}}, \tag{3}$$

we then obtain the well-known relation

$$\frac{d\,\sigma_{\gamma N \to \psi N}}{d\,t}(s, t = 0) = \frac{3\Gamma(\psi \to e^+ e^-)[s - (m_N + m_\psi)^2][s - (m_N - m_\psi)^2](1 + \rho^2)}{16\pi\alpha\,m_\psi(s - m_N^2)^2} \left(\sigma_{\psi N}^{tot}\right)^2, \tag{4}$$

where ρ is the ratio of real and imaginary parts of forward $J/\psi - N$ scattering amplitude.
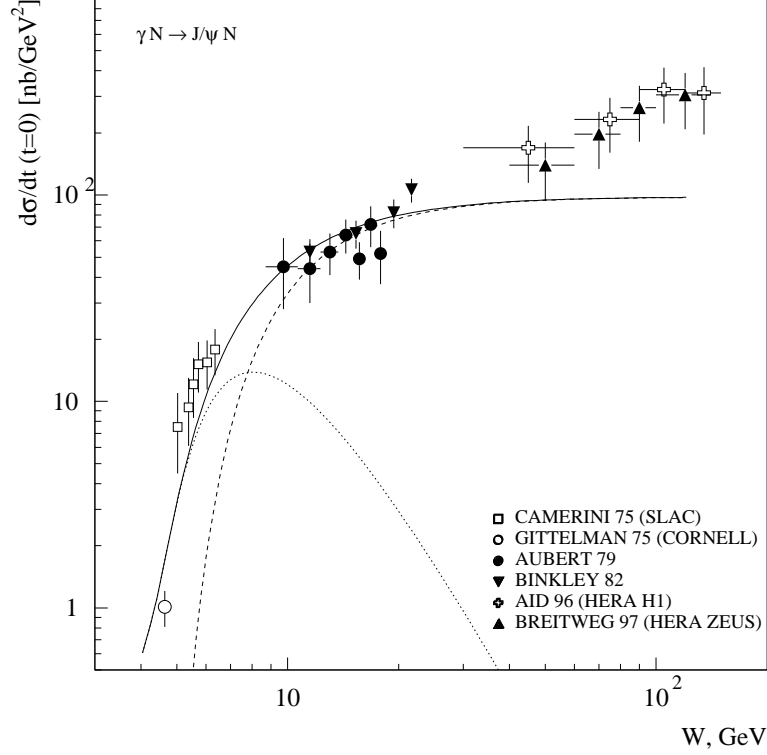
Figure 1: Forward J/ψ photoproduction data compared to our results with (solid line) and without (dashed line) the real part of the amplitude. The curves were obtained using a scaling PDF [4]

By use of the operator product expansion [2]-[5], the $J/\psi - N$ scattering amplitude in the unphysical region $\lambda \sim 0$ is obtained in terms of the nucleonic gluon distribution $g(x, \epsilon_0^2)$

$$
\mathcal{M}_{\psi N} = 2\, m_\psi \sqrt{\pi} r_0^3 \epsilon_0^2 \left(\frac{32}{3}\right)^2 \left[\int_0^1 dx \sum_{n=2,4,\ldots}^\infty x^{n-2} \left(\frac{\lambda}{\epsilon_0}\right)^n g(x, \epsilon_0^2) \times \right.
$$

$$
\times \frac{\Gamma(n + \frac{5}{2})}{\Gamma(n+5)} {}_3F_2 \left(\frac{5}{4} + \frac{n}{2}, \frac{7}{4} + \frac{n}{2}, 1+n; \frac{(5+n)}{2}, 3 + \frac{n}{2}; -\frac{m_N^2}{4\epsilon_0^2} x^2\right)
$$

$$
\left. - \frac{m_N^2}{4\epsilon_0^2} \frac{\Gamma(\frac{9}{2})}{\Gamma(7)} \int_0^1 dx\, g(x, \epsilon_0^2) \, {}_3F_2 \left(1, \frac{9}{4}, \frac{11}{4}; \frac{7}{2}, 4; -\frac{m_N^2}{4\epsilon_0^2} x^2\right) \right]. \tag{5}
$$

The quantities $r_0 = 4/(3\alpha_s m_c)$ and $\epsilon_0 = m_c (3\alpha_s/4)^2$ correspond to the 'Bohr' radius and the 'Rydberg' energy of the lowest $c\bar{c}$ bound state J/ψ, with m_c for the mass of the charm quark. The gluon distribution $g(x, \epsilon_0^2)$ is renormalized at the quarkonium binding energy scale ϵ_0. Eq. (5) includes target mass corrections [5]; neglecting these would lead back to the form used in [4]. This form encounters problems in calculating the real part and hence would here give an incorrect threshold behavior.

Taking into account the obvious analytical properties of the amplitude (5), one now relates the physical to the unphysical regions in λ; together with the optical theorem (3)
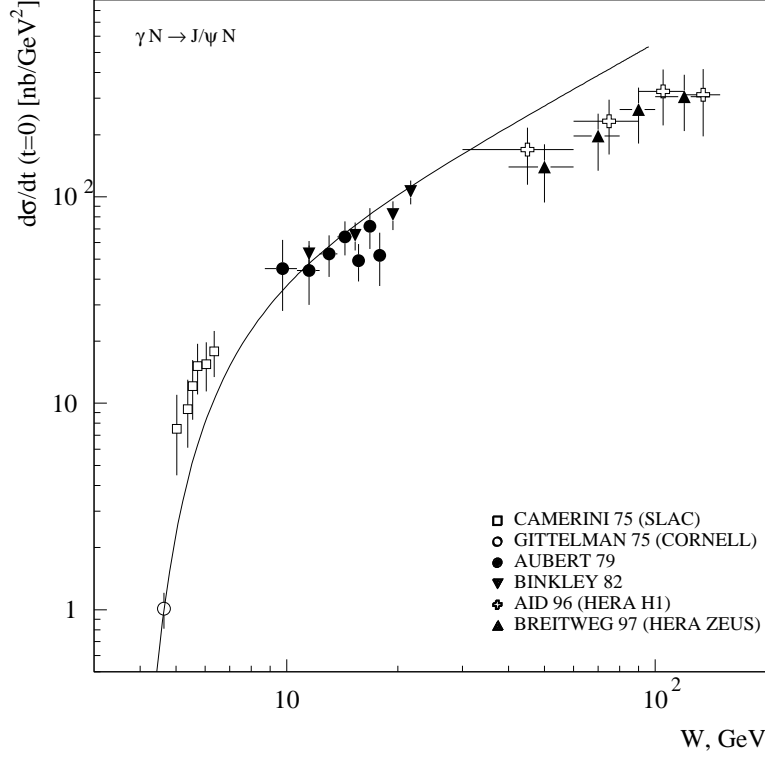
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Figure 2: The same as in Fig.1, but with the curves obtained using the PDF MRS H.

this leads to sum rules for the $J/\psi - N$ cross section,

$$\int_0^1 dy\, y^{n-2}(1-y^2)^{1/2}\sigma_{\psi N}^{tot}(m_N/y) =$$

$$I(n)\int_0^1 dx\, x^{n-2}\, g(x, M_q^2)\, {}_3F_2\left(\frac{5}{4}+\frac{n}{2}, \frac{7}{4}+\frac{n}{2}, 1+n; \frac{(5+n)}{2}, 3+\frac{n}{2}; -\frac{m_N^2}{4\epsilon_0^2}x^2\right), \quad (6)$$

where $y = m_N/\lambda$ and $I(n) = (\pi^{3/2}/2)(32/3)^2[\Gamma(n+5/2)/\Gamma(n+5)]r_0^3\epsilon_0(m_N/\epsilon_0)^{n-1}$. In a first iteration, the solution of these sum rules can be written as a convolution of the gluon distribution function and the gluon-J/ψ cross section (see [3]),

$$\sigma_{N\psi}^{(0)}(\lambda) = \frac{8\pi}{9}\left(\frac{32}{3}\right)^2\frac{1}{\alpha_S m_c^2}\int_{\epsilon_0/\lambda}^1 dx\frac{((x\lambda/\epsilon_0)-1)^{3/2}}{(x\lambda/\epsilon_0)^5}\frac{g(x, M_q^2)}{x}, \quad (7)$$

neglecting terms of order $m_N^2/\epsilon_0{}^2$. Including higher order terms, the full solution can be obtained iteratively, each step providing a contribution from the corresponding term of the hypergeometric series. Effectively, these target mass corrections change the x-variable in the convolution (7) and thus the resulting threshold behaviour.

For a given gluon distribution $g(x, \epsilon_0)$, we thus obtain $\sigma_{\psi N}^{tot}$ and hence the imaginary part of the $J/\psi - N$ scattering amplitude. To determine the forward J/ψ-photoproduction cross section (see Eq's. (1) and (2)), we need the real part of the amplitude as well. The high energy behaviour of the corresponding amplitude makes it possible to express this
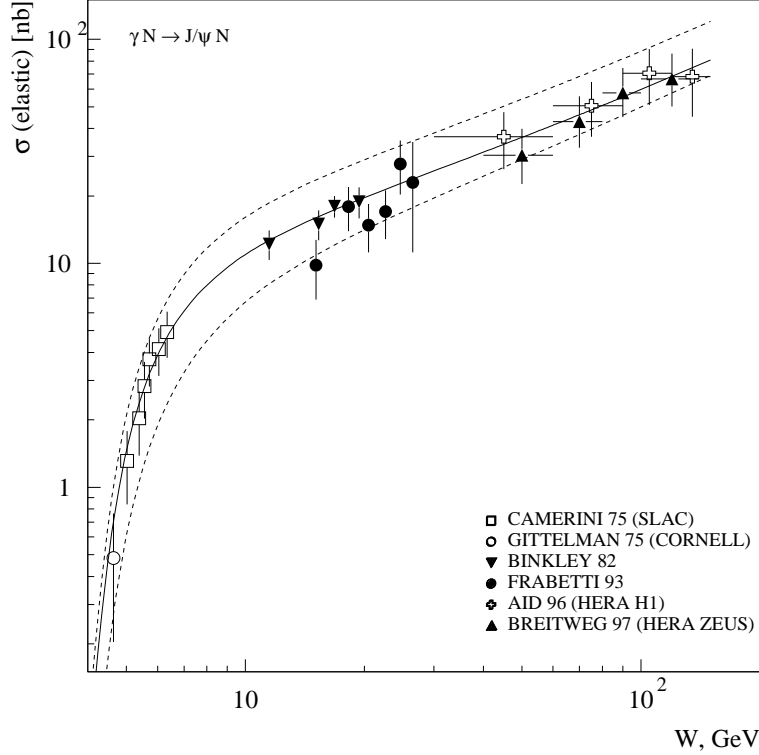
Figure 3: Data for the elastic J/ψ-photoproduction cross section, compared to the prediction obtained with the PDF MRS H.

in terms of dispersion integrals with one subtraction, performed e.g. at $\lambda = 0$ [6, 7]:

$$Re\,\mathcal{M}_{\psi N}(\lambda) = \mathcal{M}_{\psi N}(0) + \frac{2\,\lambda^2}{\pi} \int_{\lambda_0}^{\infty} \frac{d\,\lambda'}{\lambda'} \frac{Im\,\mathcal{M}_{\psi N}(\lambda')}{\lambda'^2 - \lambda^2}. \tag{8}$$

The subtraction constant $\mathcal{M}_{\psi N}(0)$, which is needed to estimate the behaviour of the real part of the amplitude near threshold is obtained by calculating the corresponding limit $(\lambda \to M_\psi)$ in Eq. (5). This is a self-consistent approach if we are given the analytical expression Eq. (5) for the amplitude. A more phenomenological and perhaps also more realistic procedure would be to use the low-energy theorems to estimate the subtraction constant, following some recent work [8].

To complete our calculation, we have to specify the gluon distribution function of the nucleon and fix the overall normalization in terms of the different constants in Eq. (4) and related quantities. For $g(x, m_c^2)$, we use two parametrizations specified in [4] and [9]. The MRS H of [9] parametrization, in particular, takes into account the recent HERA results at small x and thus seems preferable for our analysis. Although all quantities in our formulae are in a sense 'physical' constants, uncertainties enter through the c-quark mass m_c and the resulting values of α_s and ϵ_0; an addition, both the VMD model and the Coulombic description of the J/ψ may require corrections. We therefore treat the overall normalization of our result as an open constant, as was also done in [3].

We now want to compare our results with the data available from low energy Cornell-SLAC [10] up to recent high energy HERA studies [11]. These experiments measure

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the photoproduction cross section as function of the invariant momentum transfer t in the 'diffractive' region at small t. One can now fit the t-dependence in the customary exponential form $\exp(-bt)$ and then either extrapolate it to the unmeasurable limit $t = 0$ at which our prediction (1) holds, or integrate over t from $t_0 = t_{\max}(s)$ to infinity to obtain the 'elastic' photoproduction cross section $\sigma_{\gamma N \to J/\psi N}(\text{elastic})$.

Figs. 1 gives the c.m.s. energy dependence of the forward differential cross section for the scaling PDF-parameterization $g(x) = 2.5(1 - x)^4$ used in [4]; we show both our complete result and the form obtained by neglecting the real part, as in [4]. It is seen that the inclusion of the real part greatly improves the threshold behaviour; the agreement is now quite good, except for the high energy data, for which the small x behaviour of the PDF becomes important. Since the PDF form used here does not include this, it is clear that there will be deviations at high W. In Fig. 2, we then show the corresponding result for the new PDF MRS H, which does include the small x results from Hera. While the qualitative agreement is reasonable, there are definite deviations; these would become weaker for a less singular small x form of the PDF. Moreover, there is some experimental discrepancy between t-dependence of the SLAC data compared to other data in the same energy region; we shall return to this shortly. It is also not clear if a fine-tuning of m_c and α_S would improve the situation.
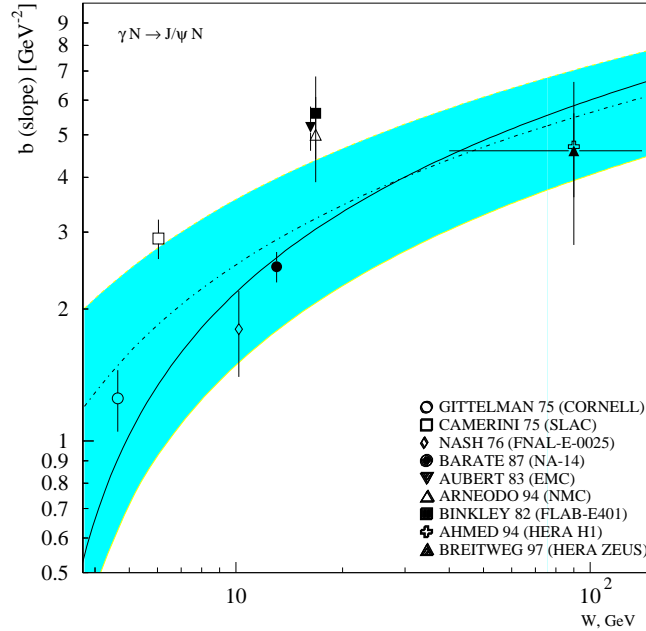

Figure 4: The energy dependence of the slope parameter for differential elastic J/ψ-photoproduction. The solid line corresponds to a best fit with $b = a_0 + a_1 \ln W^2$, giving $a_0 = -1.64 \pm 0.26$ and $a_1 = 0.83 \pm 0.06$; for data references, see [11]

In Fig. 3, we then show our results for the exclusive J/ψ photoproduction cross section as a function of W, compared to relevant experimental data; the theoretical error range is explained below. The curve was obtained for the MRS H parameterization of the PDF,

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using the approach mentioned above. Thus, we integrate the forward differential cross section in the diffractive peak up to t_{max} [12] with the slope parameter b as logarithmic function of W ('diffraction cone shrinkage'). This integrated result is much less sensitive to the parameter values involved and provides a good interpolation between the high energy (small x) data and the near-threshold behaviour of the cross section.

Finally, we want to return shortly to the slope parameter parametrisation we have used above. In Fig. 4 we show the result of fitting the form $b = a_0 + a_1 \log W^2$ to all available measurements; a best fit gives $a_0 = -1.64 \pm 0.26$ and $a_1 = 0.83 \pm 0.06$. It is seen that the SLAC data give a slope parameter considerably higher than that of the other experiments in the same energy region. This leads directly to the comparatively high values of $d\sigma/dt(t = 0)$, which are obtained by an extrapolation to $t = 0$ using the measured slope parameter. – The energy variation of b indicates the presence of contributions from 'soft' as well as from a 'hard' Pomeron [14]. Here the role of the low energy points is crucial; a restriction to only high energy points [13] could lead to an energy-independent slope as expected for a 'hard' Pomeron.

In conclusion, we note that our analysis of photoproduction confirms the relation between the energy dependence of the cross section and the x-dependence of the gluon distribution function of the nucleon [4]. This relation was derived for J/ψ-hadron interactions and enters photoproduction through VMD. The success of such a consistent treatment of these two reactions does not support expectations about inherently different behaviour [13].

Acknowledgements

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